The Tolman "Antitelephone" Paradox: Its Solution by Tachyon Mechanics [(*)]

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Abstract – Some recent experiments led to the claim that something can travel faster than light in vacuum. However, such results do not seem to place relativistic causality in jeopardy. Actually, it is possible to solve also the known causal paradoxes, devised for "faster than c" motion: even if this is not widely recognized. Here we want to show, in detail and rigorously, how to solve the oldest causal paradox, originally proposed by Tolman, which is the kernel of so many further tachyon paradoxes. The key to the solution is a careful application of *tachyon mechanics*, that can be unambiguously derived from special relativity.

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Introduction. – Some recent experiments, performed at Cologne([1]), Berkeley([2]) and Florence([3]) led to the claim that *something* can travel with a speed larger than the speed c of light in vacuum, thus confirming some older predictions.([4]) Nevertheless, such results do not seem to place relativistic causality in jeopardy.

Actually, it is possible to solve also the known causal paradoxes, devised for "faster than light" motion, even if this is not widely recognized.

In fact, claims exist since long([5]) that all the ordinary causal paradoxes proposed for tachyons can be solved (at least "in microphysics"([6])) on the basis of the "switching procedure" (swp) introduced by Stückelberg([7]), Feynman([7]) and Sudarshan([5]), also known as the reinterpretation principle: a principle which in refs.([8,9]) has been given the status of a fundamental postulate of special relativity (SR), both for bradyons [slower–than–light particles] and for tachyons. Schwartz,([10]) at last, gave the swp a formalization in which it becomes "automatic".

However, the effectiveness of the swp and of those solutions is often overlooked, or misunderstood. Here we want therefore to show, in detail and rigorously, how to solve the oldest "paradox", i.e. the *antitelephone* one, originally proposed by Tolman([11]) and then reproposed by many authors. We shall refer to its recent formulation by Regge,([12]) and spend some care in solving it, since it is the kernel of many other paradoxes. Let us stress that: (i) any careful solution of the tachyon causal "paradoxes" has to make recourse to explicit calculations based on the mechanics of tachyons; (ii) such tachyon mechanics can be unambiguously and univocally derived from SR, by referring the Superluminal objects to the class of the ordinary, subluminal observers *only* (i.e., without any need of introducing "Superluminal reference frames"); (iii) moreover, the comprehension of the whole subject will be substantially enhanced if one will refer himself to the (subluminal, ordinary) SR based on the *whole* proper Lorentz group $\mathcal{L}_+ \equiv \mathcal{L}_+^\uparrow \cup \mathcal{L}_+^\downarrow$, rather than on its orthochronous subgroup $\mathcal{L}_+^\uparrow$ only [see refs.([9]), and references therein]. At last, for a modern approach to the classical theory of tachyons, reference can be made to the review article([6]) as well as to refs.([13,14]).

Tachyon mechanics. – In ref.([15]) the basic tachyon mechanics can be found exploited for the processes: a) proper (or "intrinsic") emission of a tachyon T by an ordinary body A; b) "intrinsic" absorption of a tachyon T by an ordinary body A; c) exchange of a tachyon T between two ordinary bodies A and B. The word "intrinsic" refers to the fact that those processes (emission, absorption by A) are described *as they appear* in the rest-frame of A; particle T can represent both a tachyon and an antitachyon. Let us recall the following results only.

Let us first consider a tachyon moving with velocity $\boldsymbol{V}$ in a reference frame s_0. If we pass to a second frame s', endowed with velocity $\boldsymbol{u}$ w.r.t. (with respect to) frame s_0, then the new observer s' will see —instead of the initial tachyon T— an antitachyon $\overline{\mathrm{T}}$ travelling the opposite way in space (due to the swp), if and only if

$$\boldsymbol{u} \cdot \boldsymbol{V} > c^2 \,. \tag{1}$$

Recall in particular that, if $\boldsymbol{u} \cdot \boldsymbol{V} < 0$, the "switching" does *never* come into play.

Now, let us explore some of the unusual and unexpected consequences of the trivial fact that in the case of tachyons it is

$$|E| = +\sqrt{\boldsymbol{p}^2 - m_0^2} \qquad (m_0 \;\; \text{real;} \;\; \boldsymbol{V}^2 > 1) \,, \tag{2}$$

where we chose units so that, numerically, $c = 1$.

Let us, e.g., describe the phenomenon of "intrinsic emission" of a tachyon, as seen in the rest frame of the emitting body: Namely, let us consider *in its rest frame* an ordinary body A, with initial rest mass M, which emits a tachyon (or antitachyon) T endowed with (real) rest mass $m \equiv m_0$, four-momentum $p^\mu \equiv (E_{\mathrm{T}}, \boldsymbol{p})$, and velocity $\boldsymbol{V}$ along the x-axis. Let M' be the final rest mass of body A. The four-momentum conservation requires

$$M = \sqrt{\boldsymbol{p}^2 - m^2} + \sqrt{\boldsymbol{p}^2 + M'^2} \qquad \text{(rest frame)} \tag{3}$$

that is to say $[V \equiv |\boldsymbol{V}|]$:

$$2M|\boldsymbol{p}| = [(m^2 + \Delta)^2 + 4m^2M^2]^{\frac{1}{2}} \;; \quad V = [1 + 4m^2M^2/(m + \Delta)^2]^{\frac{1}{2}} \,, \tag{4}$$

where [calling $E_{\mathrm{T}} \equiv +\sqrt{\boldsymbol{p}^2 - m^2}\,$]:

(5) $$\Delta \equiv M'^2 - M^2 = -m^2 - 2ME_{\rm T} \ , \qquad \text{(emission)}$$

so that

(6) $$-M^2 < \Delta \leq -|\boldsymbol{p}|^2 \leq -m^2 \ . \qquad \text{(emission)}$$

It is essential to notice that Δ is, of course, an *invariant* quantity, which in a generic frame s writes

(7) $$\Delta = -m^2 - 2p_\mu P^\mu \ ,$$

where P^μ is the initial four-momentum of body A w.r.t. frame s.

Notice that in the generic frame s the process of (intrinsic) emission can appear either as a T emission or as a $\overline{\rm T}$ absorption (due to a possible "switching") by body A. The following theorem, however, holds:([15])

Theorem 1: << A necessary and sufficient condition for a process to be a tachyon emission in the A rest-frame (i.e., to be an *intrinsic emission*) is that during the process the body A *lowers* its rest-mass (invariant statement!) in such a way that $-M^2 < \Delta \leq -m^2$. >>

Let us now describe the process of "intrinsic absorption" of a tachyon by body A; i.e., let us consider an ordinary body A to absorb *in its rest* frame a tachyon (or antitachyon) T, travelling again with speed V along the x-direction. The four-momentum conservation now requires

(8) $$M + \sqrt{\boldsymbol{p}^2 - m^2} = \sqrt{\boldsymbol{p}^2 + M'^2} \ , \qquad \text{(rest frame)}$$

which corresponds to

(9) $$\Delta \equiv M'^2 - M^2 = -m^2 + 2ME_{\rm T} \ , \qquad \text{(absorption)}$$

so that

(10) $$-m^2 \leq \Delta \leq +\infty \,. \qquad \text{(absorption)}$$

In a generic frame s, the quantity Δ takes the invariant form

$$\Delta = -m^2 + 2p_\mu P^\mu \,. \tag{11}$$

It results in the following new theorem:

Theorem 2: << A necessary and sufficient condition for a process (observed either as the emission or as the absorption of a tachyon T by an ordinary body A) to be a tachyon absorption in the A-rest-frame —i.e., to be an *intrinsic absorption*— is that $\Delta \geq -m^2$. >>

We now have to describe the *tachyon exchange* between two ordinary bodies A and B. We have to consider the four-momentum conservation at A *and* at B; we need to choose a (single) frame relative to which we describe the whole interaction; let us choose the rest-frame of A. Let us explicitly remark, *however*, that —when bodies A and B exchange one tachyon T— the tachyon mechanics is such that the "intrinsic descriptions" of the processes at A *and* at B can a priori correspond to one of the following four cases([15]):

$$\begin{cases} 1) \quad \text{emission—absorption} \,, \\ 2) \quad \text{absorption—emission} \,, \\ 3) \quad \text{emission—emission} \,, \\ 4) \quad \text{absorption—absorption} \,. \end{cases} \tag{12}$$

Case 3) can happen, of course, only when the tachyon exchange takes place in the receding phase (i.e., while A, B are receding from each other); case 4) can happen, by contrast, only in the approaching phase.

Let us consider here only the particular tachyon exchanges in which we have an "intrinsic emission" at A, and in which moreover the velocities $\boldsymbol{u}$ of B and $\boldsymbol{V}$ of T w.r.t.

body A are such that $\boldsymbol{u} \cdot \boldsymbol{V} > 1$. Because of the last condition and the consequent "switching" (cf. Eq.(1)), from the rest-frame of B one will therefore observe the flight of an antitachyon $\overline{\mathrm{T}}$ *emitted* by B and absorbed by A (the *necessary* condition for this to happen, let us recall, being that A, B *recede* from each other).

More generally, the kinematical conditions for a tachyon to be exchangeable between A and B can be shown to be the following:

I) Case of "intrinsic emission" at A:

$$\begin{cases} \text{if } \boldsymbol{u} \cdot \boldsymbol{V} < 1 \,, \quad \text{then } \Delta_{\mathrm{B}} > -m^2 \quad (\longrightarrow \text{intrinsic absorption at B}); \\ \\ \text{if } \boldsymbol{u} \cdot \boldsymbol{V} > 1 \,, \quad \text{then } \Delta_{\mathrm{B}} < -m^2 \quad (\longrightarrow \text{intrinsic emission at B}). \end{cases} \tag{13}$$

II) Case of "intrinsic absorption" at A:

$$\begin{cases} \text{if } \boldsymbol{u} \cdot \boldsymbol{V} < 1 \,, \quad \text{then } \Delta_{\mathrm{B}} < -m^2 \quad (\longrightarrow \text{intrinsic emission at B}); \\ \\ \text{if } \boldsymbol{u} \cdot \boldsymbol{V} > 1 \,, \quad \text{then } \Delta_{\mathrm{B}} > -m^2 \quad (\longrightarrow \text{intrinsic absorption at B}). \end{cases} \tag{14}$$

Now, let us finally pass to examine the Tolman paradox.

The paradox. – In Figs.1,2 the axes t and t' are the world-lines of two devices A and B, respectively, which are able to exchange tachyons and move with constant relative speed u, $[u^2 < 1]$, along the x-axis. According to the terms of the paradox (Fig.1), device A sends tachyon 1 to B (in other words, tachyon 1 is supposed to move forward in time w.r.t. device A). The device B is constructed so as to send back tachyon 2 to A as soon as it receives tachyon 1 from A. If B has to *emit* (in its rest-frame) tachyon 2, then 2 must move forward in time w.r.t. device B; that is to say, the world-line BA_2 must have a slope *lower* than the slope BA' of the x'-axis (where $\mathrm{BA}'//x'$): this means that A_2 must stay above A'. If the speed of tachyon 2 is such that A_2 falls between A' and A_1, it seems that 2 reaches A (event A_2) *before* the emission of 1 (event A_1). This appears to realize an *anti-telephone*.

The solution. – First of all, since tachyon 2 moves backwards in time w.r.t. body A, the event A_2 will appear to A as the emission of an antitachyon $\overline{2}$. The observer "t" will see his own device A (able to exchange tachyons) emit successively towards B the antitachyon $\overline{2}$ and the tachyon 1.

At this point, some supporters of the paradox (overlooking tachyon mechanics, as well as relations (12)) would say that, well, the description put forth by the observer "t" can be orthodox, but then the device B is no longer working according to the stated programme, because B is no longer emitting a tachyon 2 on receipt of tachyon 1. Such a statement would be wrong, however, since the fact that " t " observes an "intrinsic emission" at A_2 *does not mean* that " t' " will see an "intrinsic absorption" at B! On the contrary, we are just in the case considered above, between eqs. (12) and (13): intrinsic emission by A, at A_2, with $\boldsymbol{u} \cdot \boldsymbol{V}_{\overline{2}} > c^2$, where $\boldsymbol{u}$ and $\boldsymbol{V}_{\overline{2}}$ are the velocities of B and $\overline{2}$ w.r.t. body A, respectively; so that *both* A *and* B experience an intrinsic *emission* (of tachyon 2 or of antitachyon $\overline{2}$) in their own rest frame.

But the tacit premises underlying the "paradox" (and even the very terms in which it was formulated) were "cheating" us *ab initio.* In fact, Fig.1 makes it clear that, if $\boldsymbol{u} \cdot \boldsymbol{V}_{\overline{2}} > c^2$, then for tachyon 1 *a fortiori* $\boldsymbol{u} \cdot \boldsymbol{V}_1 > c^2$, where $\boldsymbol{u}$ and $\boldsymbol{V}_1$ are the velocities of B and 1 w.r.t. body A. Therefore, due to the previous consequences of tachyon mechanics, observer "t'" will see B intrinsically *emit* also tachyon 1 (or, rather, antitachyon $\overline{1}$). In conclusion, the proposed chain of events does *not* include any tachyon absorption by B (in its rest frame).

For body B to *absorb* (in its own rest frame) tachyon 1, the world-line of 1 ought to have a slope *higher* than the slope of the x'-axis (see Fig.2). Moreover, for body B to *emit* ("intrinsically") tachyon 2, the slope of the of 2 should be lower than the x'-axis'. In other words, when the body B, programmed to emit 2 as soon as it receives 1, does actually do so, the event A_2 does happen *after* A_1 (cf. Fig.2), as requested by causality.

The moral. – The moral of the story is twofold: i) one should never *mix* the descriptions (of one phenomenon) yielded by different observers; otherwise —even in ordinary physics— one would immediately meet contradictions: in Fig.1, e.g., the motion direction of 1 is assigned by A and the motion-direction of 2 is assigned by B; this is "illegal"; ii) when proposing a problem about tachyons, one must comply([5]) with the rules of tachyon mechanics([15]); this is analogous to complying with the laws of *ordinary* physics when formulating the text of an *ordinary* problem (otherwise the problem in itself will be "wrong").

Most of the paradoxes proposed in the literature suffered the above shortcomings.

Notice once more that, in the case of Fig.1, neither A nor B regard event A_1 as the cause of event A_2 (or *vice-versa*). In the case of Fig.2, on the other hand, both A and B consider event A_1 to be the cause of event A_2: but in this case A_1 does chronologically precede A_2 according to both observers, in agreement with the relativistic covariance of the law of retarded causality.

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Figure captions:

Fig.1 – The apparent chain of events, according to the terms of the paradox.

Fig.2 – Solution of the paradox: see the text.

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